THEORETICAL STUDIES OF TURBULENCE IN PLASMAS

under the direction of

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The following topics have been investigated during the third six-month period:

1. Stochastic acceleration

2. Particle acceleration in astrophysics

3. A classification of instabilities

1. Stochastic Acceleration (P. A. Sturrock)

The theory of stochastic acceleration, which was discussed in Semi-annual Status Report No. 2, has been developed in more detail in the nonrelativistic approximation. For simplicity, acceleration parallel and transverse to the magnetic field have been considered separately. It has been shown that, in the "weak-field" approximation, the acceleration process may be described by a Fokker-Planck equation. The coefficients of this equation are expressible in terms of the correlation functions for the electric field. In certain cases, these coefficients may be expressed in terms of the energy spectrum of the field.

The relationship of this theory to the quasi-linear theory of instabilities and to the phenomenon of Landau damping has been investigated. Since the physical assumptions of the present theory of stochastic acceleration are simple and direct, it is believed that this connection helps to clarify the nature of the quasilinear theory and of Landau damping.

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It has been found that, in the absence of any other effect (such as "loading" of the accelerating field by the accelerated particle) the transverse energy distribution tends towards a Maxwellian form with a temperature which increases linearly with time. The same is true for longitudinal acceleration if the spectrum of the electric field is flat over the range of phase velocities of interest.

The application of this acceleration process to the earth's bow shock has been considered, and it has been found that the high energy electrons observed in the transition region may have been accelerated by quite weak random or quasi-random electric fields.

For other astrophysical applications, it will be necessary to develop a relativistic theory of stochastic acceleration.

An account of this work has been prepared as SUIPR No. 21.[1]

2. Particle Acceleration in Astrophysics (P. A. Sturrock and D. E. Hall)

The presence of highly suprathermal particles is a well-known phenomenon in astrophysics; prominent examples are to be found in supernova remnants, radio galaxies, cosmic rays, and solar radio bursts and the particle ejections sometimes associated with them. The acceleration of these particles, ions as well as electrons, has not been satisfactorily explained in any of these contexts. It seems likely that these examples, even though they are of very different scales, may be related by a common basic mechanism. Since the solar events offer the greatest possibility for discriminating among different ideas by recourse to observations, we hope that a study directed specifically at these events may still prove the best key to understanding the general phenomenon.

The theory of particle acceleration which has dominated astrophysical thinking of this problem for the past two decades is that of Fermi acceleration. This is a particular form of stochastic acceleration, which is one of the principal items of study under this program.

However, it is not certain that cosmic rays and high-energy electrons are indeed accelerated by the Fermi mechanism, or even that the mechanism is intrinsically stochastic, although it must be so to some extent. For this reason, the present project has been renamed "Particle Acceleration in Astrophysics" to emphasize that one should study the available data for guidance, and that one should keep an open mind as to whether or not the acceleration mechanism (or mechanisms) will turn out to be stochastic.

The previous report described in their primitive form two ideas on this subject which we wished to pursue. One of these was that a solar flare might produce a sudden wrenching of a tube of magnetic flux, and that if the tube extended into regions of much lower particle density an Alfven "shock wave" travelling outward along it might produce acceleration in a manner roughly analogous to a whiplash. This idea has lain dormant and is just now receiving more detailed study.

The other idea was that of two-stage acceleration, and it has been the center of our attention during this period. This was prompted by the theory of Sturrock and Coppi,[2] in which it is predicted that an electric field parallel to the magnetic field is generated by the instability responsible for the flash phase of a solar flare. The estimated strength of this field was such that appreciable runaway acceleration of electrons might take place, but much too small for any such thing to happen to ions. We proposed that runaway electrons, particularly if following field lines which lead to regions of much lower density, might transfer appreciable energy to a few favored ions through strong charge-separation fields. This is a one-dimensional picture, but collective and non-linear, so it is practical to study it by means of a computer model.

We have developed a computer program for this purpose in which the plasma is represented by two families of charge sheets moving in one dimension under the influence of their self-consistent electric field. The ion-to-electron charge and mass ratios may be varied independently. The sheets are advanced, ordinarily around ten time per plasma period, according to the equations

$$v(t + dt) = v(t) + a(x(t),t)\, dt,$$

$$x(t + dt) = x(t) + v(t + dt)\, dt.$$

These reduce to the same correct form when $dt \to 0$ as does the more intuitive set which uses $v(t)$ on the right-hand side of the second equation, but for finite dt our choice eliminates an instability.

The simple form of the Coulomb field in one dimension means that $a(x,t)$ may always be found merely by knowing the net charge imbalance on either side of x at time t; the distance at which the other sheets lie does not matter. Now the desired model must make a clear distinction between runaway velocities and unavoidable noise from the finite-grain representation, and yet have the runaways take several plasma periods to cross the region being observed; this leads to the requirement that a large number of sheets be used. The method of completely sorting the charge sheets to find the electric field consumes far too much time to be practical for more than a few hundred sheets, so we have used a method developed by Burger (Stanford Electronics Laboratories, SEL 64-012) in which the sheets are sorted only into a finite number of fixed bins. This has roughly the effect of giving the sheets a finite thickness, and should not interfere with the study of events taking place over a scale much larger than the bin size. With this method as many as 10,000 charge sheets can be advanced one step in a time of order one second on the IBM 7090.

The principal output generated by this program is an automatic plot of the space-time trajectories of any desired sampling of particles; this is a most valuable visual aid for understanding how the consequences of a given initial state will develop, and it may be supplemented whenever desired by a printout of other information. Velocity distributions in particular are informative and easily done by the machine.

Preliminary tests of several initial conditions have been made with charge ratio unity, mass ratio eight, and 1000 or fewer sheets. Whenever runaway electrons are made to approach an edge where the background density drops sharply it appears that many of the electrons are stopped and that a distinct one-sided high-energy tail is developed on the ion velocity distribution. These results are promising, and we believe that the programming as it stands is adequate for proceeding with more detailed study of this model. We consider, however, that it is necessary to parallel these detailed calculations with qualitative studies of astrophysical phenomena, to improve our judgment as to whether this type of mechanism will jibe with observations.

To this end we have been studying information about particle acceleration by solar flares. There seems to be a strong correlation of energetic-particle events with Type II and IV radio bursts rather than with Type III bursts. Since Type III bursts are associated with fast streams of electrons and Type II and IV with much slower shock waves, this must be considered a point against our two-stage acceleration model. It also seems that an energetic-particle event is more likely if a flare encroaches on the umbra of a sunspot; this point as well and the previous one favor the model based on the idea of sudden twisting impulses applied to magnetic flux tubes, mentioned at the beginning of this section. We shall continue to look for other helpful observational indications as we proceed.

3. Classification of Instabilities (P. A. Sturrock)

Since turbulence in plasmas will frequently, if not invariably, originate in an instability, it is important to consider the type of excitation of a plasma which may follow an instability.

A great deal of detailed work along these lines has been done in recent years as part of the "quasilinear" theory of the two-stream instability, and certain other instabilities. However, there is no general theorem to the effect that the electrostatic two-stream instability is typical of all instabilities. Work on solar flares and other explosive phenomena, under another contract, has indicated that instabilities may be classified into two categories, which are in some sense "explosive" and "non-explosive".

Once this question was posed, it was realized that it is closely related to studies of the stability of dynamical systems begun by Poincaré[3] and continued by Jeans[4], Lyttleton[5] and others.

In most physical systems and all astrophysical systems, one is concerned in fact with the onset of instability. That is, one must consider a system characterized by a parameter which is changing slowly in time: up to a certain value μ_c of this parameter μ, the system is supposed to follow a stable series of equilibrium states: beyond this value μ_c the equilibrium states of interest become unstable. This problem was investigated by Poincaré and others in connection with the stability of rotating liquid masses. It was discovered that when a "linear series" of equilibrium states (enumerated by the parameter μ) shows a change from stability to instability, at least two other linear series branch off from the primary series.

The simplest and most important cases are those in which just two new linear series branch off from the original one and that both series extend in the direction $\mu < \mu_c$ or both series extend in the direction $\mu > \mu_c$. These two cases are shown schematically in Fig. 1 and Fig. 2. In the first case, the two additional series must both be unstable; in the second case they must both be stable. The first case represents an explosive onset of instability, whereas the second case represents a nonexplosive onset of instability.

The important difference between these two cases is that, if the onset of instability is explosive, there will be a sudden and finite change of state by the time μ has increased beyond the value μ_c. If the onset is nonexplosive, on the other hand, the system will simply change from the original stable series to one of the two new stable series when μ takes the value μ_c, and the progress from one state to the next is determined by the rate of change of the parameter μ.

These ideas have been set out in more detail in report SUIPR No. 36.[6]

ACKNOWLEDGMENT

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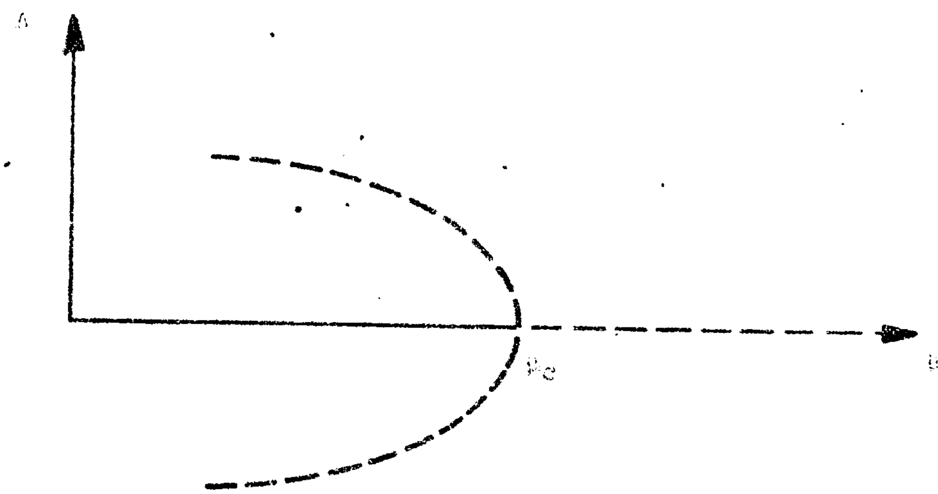

Fig. 1. Schematic representation of linear series associated with an explosive onset of instability. Solid lines represent stable states and broken lines unstable states.

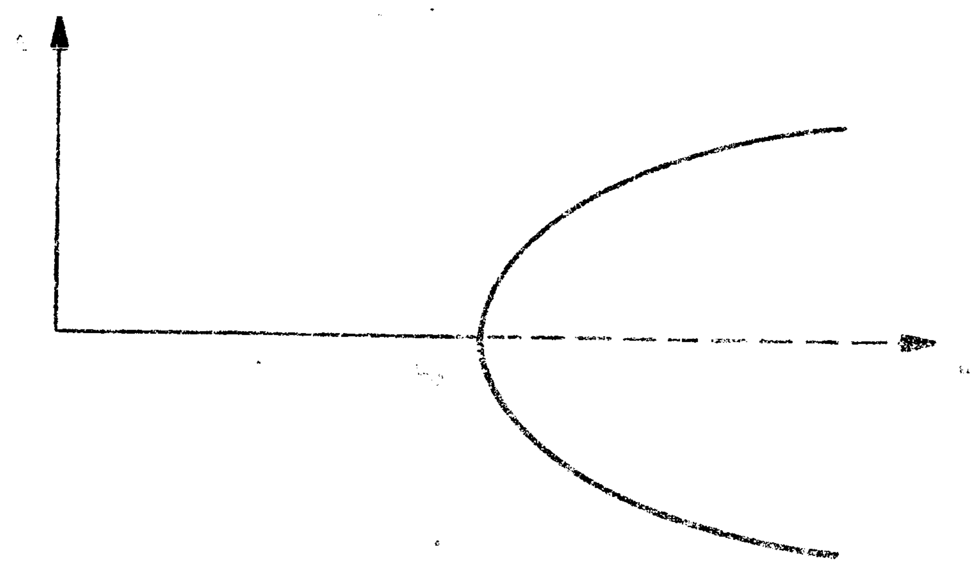

Fig. 2. [illegible] series [illegible] and [illegible]

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6. P. A. Sturrock, SUIPR 36, Stanford University, November 1965.